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The $l = 1$ Hyperfine Splitting in Bottomium as a Precise Probe of the QCD Vacuum.*

S. Titard † and F. J. Ynduráin

Departamento de Física Teórica C-XI

Universidad Autónoma de Madrid

28049 Madrid, Spain

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Abstract

By relating fine and hyperfine spittings for $l = 1$ states in bottomium we can factor out the less tractable part of the perturbative and nonperturbative effects. Reliable predictions for one of the fine splittings and the hyperfine splitting can then be made calculating in terms of the remaining fine splitting, which is then taken from experiment; perturbative and nonperturbative corrections to these relations are under full control. The method (which produces reasonable results even for the $c\bar{c}$ system) predicts a value of 1.5 MeV for the $(s = 1) - (s = 0)$ splitting in $b\bar{b}$, opposite in sign to that in $c\bar{c}$. For this result the contribution of the gluon condensate $\langle \alpha_s G^2 \rangle$ is essential, as any model (in particular potential models) which neglects this would give a

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†Electronic address: stephan@nantes.ft.uam.es.

negative $b\bar{b}$ hyperfine splitting.

14.40.Gx, 12.38.Bx, 12.38.Lg, 13.20.Gd

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I. INTRODUCTION

It has been known for a long time that the short distance strong interactions may be described by QCD in perturbation theory, and that the *leading*, short distance nonperturbative effects can be incorporated by taking into account the nonzero values of quark and gluon condensates in the physical vacuum, $|\mathrm{vac}\rangle$:

$$\langle \bar{q}q\rangle \equiv \langle \mathrm{vac}| : \bar{q}(0)q(0) : |\mathrm{vac}\rangle \neq 0 \ ,$$
$$\langle \alpha_s G^2\rangle \equiv \langle \mathrm{vac}| : G_{a\mu\nu}(0)G_a^{\mu\nu}(0) : |\mathrm{vac}\rangle \neq 0 \ .$$

From the pioneering SVZ work on QCD sum rules [1], we know that

$$\langle \alpha_s G^2\rangle = 0.042 \pm 0.020 \quad \mathrm{GeV}^4 \ , \tag{1.1}$$

a value confirmed by subsequent analyses. These methods are applicable to study bound states of heavy quarks, as shown in the papers of Leutwyler [2] and Voloshin [3] and, more recently, in our work [4,5] where we have demonstrated that, indeed, a consistent description of $n = 1$ $c\bar{c}$ states and $n = 1$, 2 $b\bar{b}$ ones (n being the principal quantum number) is obtained if one includes perturbative corrections in the form of radiative corrections to the Coulombic, short distance QCD potential,

$$\frac{-C_F\alpha_s}{r} \ , \ C_F = 4/3 \tag{1.2}$$

as well as nonperturbative ones through the contributions of $\langle \alpha_s G^2\rangle$ (the quark condensate contributes a negligible amount). In particular in Ref. [5] we found the following values for the fine and hyperfine splittings in bottomium, with $n = 2$, $l = 1$, s the total spin, and j the total angular momentum:

$$\Delta_{21} \equiv M(\chi_b(j = 2)) - M(\chi_b(j = 1)) = 21 \ {}^{+3}_{-6} \mp 2 \ \mathrm{MeV}$$
$$\Delta_{10} \equiv M(\chi_b(j = 1)) - M(\chi_b(j = 0)) = 29 \ {}^{+5}_{-9} \mp 3 \ \mathrm{MeV} \tag{1.3}$$
$$\Delta_{\mathrm{hf}} \equiv M_{\mathrm{average}}(\chi_b(s = 1, l = 1)) - M(\chi_b(s = 0, l = 1)) = 1.5 \mp 0.5 \pm 0.5 \ \mathrm{MeV} \ . \tag{1.4}$$

Δ_{21} and Δ_{10} may be compared to the experimental figures

$$\Delta_{21}^{\rm exp} = 21 \pm 1\ {\rm MeV},\ \ \Delta_{10}^{\rm exp} = 32 \pm 2\ {\rm MeV}\,. \tag{1.5}$$

In Eqs. (1.3) and (1.4) the first error is due to that in the QCD parameter Λ,

$$\Lambda(4\ {\rm flavours}, 2\ {\rm loops}) = 200\ \ {}^{+80}_{-60}\,{\rm MeV}\ , \tag{1.6}$$

and the the second to that of $\langle \alpha_s G^2 \rangle$ as given by Eq. (1.1).

It is remarkable that the prediction of $\Delta_{\rm HF}$ for $b\bar{b}$ suggests a value *opposite* in sign to that of $\bar{c}c$ (where experimentally, $\Delta_{\rm HF}^{\rm exp}(\bar{c}c) = -0.9$ MeV). This change of sign is due to the structure of the QCD vacuum through the contribution of the gluon condensate. In fact, and as we will show below, any calculation neglecting this would give a *negative* $\Delta_{\rm HF}$, of the order of -1 to -2 MeV. Thus, a measurement of $\Delta_{\rm HF}$ can be directly interpreted as a measurement of $\langle \alpha_s G^2 \rangle$.

The results reported above, Eqs. (1.3) and (1.4) are less impressive than what the agreement with experiment would lead one to believe. The reason is that they contain radiative and nonperturbative contributions which are of relative order unity, thus impairing the reliability of the calculation. In this note, however, we show that by *combining* the fine (Eq. (1.3)) and hyperfine (Eq. (1.4)) splittings one can get a clean prediction for the last, in which both radiative and nonperturbative effects are small and fully under control.

II. RADIATIVE AND NONPERTURBATIVE INTERACTIONS.

As shown by several people (cf. Refs [4,5] for details and references) the leading perturbative, radiative and nonperturbative interactions that contribute to the fine and hyperfine splittings are the LS, T (tensor) and HF (hyperfine) potentials,

$$\begin{aligned} V_{\rm LS}(\vec{r}) &= \frac{3C_F\alpha_s(\mu^2)}{2m^2r^3}\,\vec{L}\cdot\vec{S}\,\left\{1+\left[\frac{\beta_0}{2}(\ln r\mu-1)+2(1-\ln mr)+\frac{125-10\,n_f}{36}\right]\frac{\alpha_s}{\pi}\right\} && (2.1)\\ &\equiv V_{\rm LS}^{(0)}+V_{\rm LS}^{(\rm rad)}\ ,\ V_{\rm LS}^{(0)} = \frac{3C_F\alpha_s(\mu^2)}{2m^2r^3}\,\vec{L}\cdot\vec{S}\ , \\ V_{\rm T}(\vec{r}) &= \frac{C_F\alpha_s(\mu^2)}{4m^2r^3}\,\left\{1+\left[D+\frac{\beta_0}{2}\ln r\mu-3\ln mr\right]\frac{\alpha_s}{\pi}\right\} && (2.2) \end{aligned}$$

$$\equiv V_{\rm T}^{(0)} + V_{\rm T}^{(\rm rad)}\ ,\ V_{\rm T}^{(0)} = \frac{C_F\alpha_s(\mu^2)}{4m^2r^3}\ ,$$

$$V_{\rm HF}(\vec{r}) = \left(\frac{\beta_0}{2} - \frac{21}{4}\right)\frac{C_F\alpha_s(\mu^2)}{3m^2\,r^3}\,\vec{S}^2\ ; \tag{2.3}$$

with

$$S_{12}(\vec{r}) = 2\sum_{ij} S_iS_j\left(\frac{3}{r^2}r_ir_j - \delta_{ij}\right)\ ,$$
$$\vec{S} = \vec{S}_1 + \vec{S}_2\ ,\ \vec{L} = \vec{r}\times\vec{p}\,,$$
$$C_A = 3,\ T_F = 1/2,\ \beta_0 = 11 - \frac{2\,n_f}{3},$$
$$D = \frac{4}{3}\left(3 - \frac{\beta_0}{2}\right) + \frac{65}{12} - \frac{5\,n_f}{18}.$$

$V_{\rm HF}$ has an extra piece proportional to $\delta(\vec{r})$ which however does not contribute to the states with $l = 1$ in which we are interested. A spin independent radiative correction which also intervenes indirectly is given by

$$V_{\rm SI}(r) = -\frac{C_F(a_1 + \gamma_{\rm E}\,\beta_0/2)}{\pi r}\alpha_s^2 - \frac{C_F\beta_0\alpha_s^2\ln r\mu}{2\pi r}\ ;\ a_1 = \frac{31\,C_A - 20\,n_fT_F}{36}\,. \tag{2.4}$$

The nonperturbative interactions are generated by a term

$$-\,g\vec{r}\cdot\vec{\underset{\sim}{\mathcal{E}}} + \frac{g}{2m^2}(\vec{S}\times\vec{p})\cdot\vec{\underset{\sim}{\mathcal{E}}} - \frac{g}{m}(\vec{S}_1 - \vec{S}_2)\cdot\vec{\underset{\sim}{\mathcal{B}}}\,. \tag{2.5}$$

The constant chromoelectric $\vec{\underset{\sim}{\mathcal{E}}}$ and chromomagnetic $\vec{\underset{\sim}{\mathcal{B}}}$ fields are to be taken as matrices in color space, and the vacuum is to be assumed such that

$$\langle\vec{\mathcal{E}}\rangle\ =\ \langle\vec{\mathcal{B}}\rangle\ =\ 0\ ,$$
$$\langle g^2\mathcal{B}_a^i\mathcal{B}_b^j\rangle = -\langle g^2\mathcal{E}_a^i\mathcal{E}_b^j\rangle = \frac{\pi\delta_{ij}\delta_{ab}}{3(N_c^2-1)}\langle\alpha_sG^2\rangle$$

(a, b colour indices, i, j spatial ones).

The key point in the present paper is the remark that the radial operator that appears in all three $V_{\rm LS}$, $V_{\rm T}$ and $V_{\rm HF}$ is the same one at leading order, viz., r^{-3}; and it so happens that the *largest* perturbative and nonperturbative corrections are those to the *wave functions* which are the same for all fine and hyperfine splittings. This allows us to factor these out, being left with *small* and manageable pieces.

III. FINE AND HYPERFINE SPLITTINGS.

Consider for example the fine splittings. Because the radiative pieces of $V_{\rm LS}$, $V_{\rm T}$ are to be taken in perturbation theory, and the same is true of the nonperturbative interactions given in Eq. (2.5), we find, e.g.,

$$\begin{aligned}\Delta_{21} = &\langle\Psi_{j=2}|\,(V_{\rm LS}+V_{\rm T})\,|\Psi_{j=2}\rangle - \langle\Psi_{j=1}|\,(V_{\rm LS}+V_{\rm T})\,|\Psi_{j=1}\rangle\\ &+2\left\langle\Psi_{j=2}\right|\left(-g\vec{r}\cdot\vec{\underset{\sim}{\mathcal{E}}}\right)\,\frac{1}{H^{(0)}-E^{(0)}}\,\frac{g}{2m^2}(\vec{S}\times\vec{p})\cdot\vec{\underset{\sim}{\mathcal{E}}}\left|\Psi_{j=2}\right\rangle\\ &-2\left\langle\Psi_{j=1}\right|\left(-g\vec{r}\cdot\vec{\underset{\sim}{\mathcal{E}}}\right)\,\frac{1}{H^{(0)}-E^{(0)}}\,\frac{g}{2m^2}(\vec{S}\times\vec{p})\cdot\vec{\underset{\sim}{\mathcal{E}}}\left|\Psi_{j=1}\right\rangle. \end{aligned}\tag{3.1}$$

The Ψ_j are the wave functions for the states with $n=2$, $l=1$, $s=1$ and total angular momentum j. The contributions to Δ_{21} may be split into two pieces. First we have what we may call *"external"*, $\Delta^{\rm ex}$, obtained by substituting in Eq. (3.1) the unperturbed wave functions solutions to

$$H^{(0)}\Psi_j^{(0)} = E^{(0)}\Psi_j^{(0)}\ ,$$

where the potential in $H^{(0)}$ is just the Coulombic one. It so happens that both *radiative* and *nonperturbative* contributions to $\Delta^{\rm ex}$ are *small*, at the 10% level or smaller.

The troublesome piece is what we may call *"internal"*, $\Delta^{\rm in}$, and is due to the fact that Ψ_j also contains spin–independent radiative and nonperturbative corrections:

$$\Psi_j = \Psi_j^{(0)} + \Psi_j^{\rm rad} + \Psi_j^{\rm NP}\ .$$

Then, $\Delta^{\rm in}$ would be the contribution of $\Psi_j^{\rm rad}$, $\Psi_j^{\rm NP}$ to Eq. (3.1) (the radiative corrections are caused by the spin–independent corrections to the potential, and the non perturbative ones by the spin–independent pieces generated by Eq. (2.5), i.e, the contribution quadratic in $g\vec{r}\cdot\vec{\underset{\sim}{\mathcal{E}}}$). As stated before, however, the key point is that, when evaluating $\Delta^{\rm in}$, and because $\Psi_j^{\rm rad}$ and $\Psi_j^{\rm NP}$ are already perturbations, only the leading pieces of the potentials i.e., $V_{\rm LS}^{(0)}$, $V_{\rm T}^{(0)}$ and $V_{\rm HF}$ have to be considered, and these are all proportional to r^{-3}, hence identical for fine and hyperfine splittings.

For a precise evaluation we take the explicit formulas of Ref. [5]. Then one finds the following theoretical values for the splittings:

$$\begin{aligned}
\Delta_{10}^{\rm th} &= \frac{5}{4}\,(1+\delta_{\rm rad})\,\Delta_{21}^{\rm th}\;-\delta_{\rm NP}\,,\\
\delta_{\rm rad} &= \left[\frac{3}{4}\ln\frac{C_F\widetilde{\alpha}_s}{2}+\frac{80+13n_f}{96}+\frac{3}{4}\gamma_{\rm E}\right]\frac{\alpha_s}{\pi}\,,\\
\delta_{\rm NP} &= \frac{2244}{3315}\,\frac{\pi\langle\alpha_s G^2\rangle}{m^3(C_F\widetilde{\alpha}_s)^2}\,,\\
\widetilde{\alpha}_s &= \left[1+\frac{\gamma_{\rm E}\beta_0/2+(93-10n_f)/36}{\pi}\alpha_s\right]\,\alpha_s\,.
\end{aligned} \tag{3.2}$$

As for the hyperfine splitting

$$\Delta_{\rm HF}^{\rm th} = \frac{5}{24}\left(\frac{\beta_0}{2}-\frac{21}{4}\right)\,C_F\alpha_s\Delta_{21}^{\rm th}+\frac{976}{1053}\,\frac{\pi\langle\alpha_s G^2\rangle}{m^3(C_F\widetilde{\alpha}_s)^2}\,. \tag{3.3}$$

The nonperturbative piece of Eq. (3.3) is due to the term $-\dfrac{g}{m}(\vec{S}_1-\vec{S}_2)\cdot\underset{\sim}{\vec{\mathcal{B}}}$ in Eq. (2.5). In Ref. [5], the best overall fit to $n=2$ states was obtained for $\alpha_s(\mu^2)=0.38$ (this corresponds to μ=0.93 GeV). In this paper we will allow $\alpha_s(\mu^2)$ to vary between 0.33 and 0.43 which corresponds to the range $2\,\text{GeV}\geq\mu\geq 0.8$ GeV, the expected "relevant" scale, $\mu\sim\langle\vec{k}^2\rangle_{21}^{1/2}$. In all this range $|\delta_{\rm rad}|\lesssim$ 10%, and $|\delta_{\rm NP}|\lesssim$ 5%: we check that both radiative and nonperturbative corrections to the fine splittings, Eq. (3.2), are small. Agreement with experiment is excellent in all the range. This is shown is Fig. 1 where we plot the values of $\Delta_{21}^{\rm th}$, $\Delta_{10}^{\rm th}$ that follow from Eq. (3.2) by treating, in Eq. (3.2), $\Delta_{21}^{\rm th}$ as a free parameter, then fitting $\Delta_{21}^{\rm th}$, $\Delta_{10}^{\rm th}$ to experiment. For all the range, agreement between the theoretical values $\Delta^{\rm th}$ and experimental ones (cf. Eq. (1.5)) is better than 10% with respect to central experimental values, and agreement within experimental errors is even obtained for $\alpha_s=0.43$.

IV. DISCUSSION AND RESULTS.

Allowing $\alpha_s=0.38\pm0.05$, and the range of Eq. (1.1) for $\langle\alpha_s G^2\rangle$, Eqs. (3.2) and (3.3) yield a very reliable prediction for the hyperfine splitting (see Fig. 2):

$$\Delta_{\rm HF}=1.5\;{}^{+0.8}_{-1.2}\pm0.5\ \text{MeV} \tag{4.1}$$

(the first error is due to the variation of α_s, and the second to the error in $\langle\alpha_s G^2\rangle$).

A few words should be said on this. As Eq. (3.3) shows, $\Delta_{\rm HF}$ is the sum of two terms, a perturbative and a nonperturbative one. That the second one dominates is due to the fact that the perturbative contribution itself is the *difference* between two pieces, proportional respectively to $\beta_0/2$ and $21/4$, each one large, but which cancel almost exactly: for $n_f=4$, $\beta_0/4=4.17$, while $21/4=5.25$. And the whole perturbative term is still smaller because the *tree level* potential is proportional to $\delta(\vec{r})$, hence gives zero for $l=1$ states. Thus an effect potentially $\mathcal{O}(\beta_0/2)\sim 4$ is actually of order $(\beta_0/2-21/4)\,\alpha_s/\pi\sim -0.13$. This is very much suppressed and thus highlights by contrast the nonperturbative contribution.

A remarkable feature of the splitting (4.1) is that it *cannot* be reproduced by the use of any of the phenomenological potentials available on the market. Indeed, *any* model that neglects the nonzero expectation value of $\vec{\underset{\sim}{\mathcal{B}}}$ in the QCD vacuum will *necessarily* yield a *negative* $\Delta_{\rm HF}$. In particular, if one pretends to simulate nonperturbative effects by use of spin–independent potentials, then one has Eqs. (3.2) and (3.3) replaced by:

$$\begin{aligned}\Delta_{10}^{\rm pot} &= \frac{5}{4}\,(1+\delta_{\rm rad})\,\Delta_{21}^{\rm pot}\\ \Delta_{\rm HF}^{\rm pot} &= \frac{5}{24}\left(\frac{\beta_0}{2}-\frac{21}{4}\right)C_F\alpha_s\Delta_{21}^{\rm pot}\,.\end{aligned}\tag{4.2}$$

A simple calculation, also for the range $\alpha_s=0.38\pm0.05$ gives good agreement for $\Delta_{10}^{\rm pot}$, $\Delta_{21}^{\rm pot}$ with experiment, but now

$$\Delta_{\rm HF}^{\rm pot}=-1.2\ {}^{+0.3}_{-0.4}\ {\rm MeV}\,.\tag{4.3}$$

The gap between Eq. (4.3) and Eq. (4.1) is sufficiently large that a measurement, probably feasible with b–factories, should be able to reveal it.

As a last comment, let us remind the reader that the analysis we have carried is justified only at short distances. For $b\bar{b}$ with $n=2\,,l=1$, $\langle r\rangle_{21}\sim(1\ {\rm GeV})^{-1}$. For $t\bar{t}$ the situation is even more favourable, but the measurement is of course impossible. For $c\bar{c}$ we cannot carry a rigourous analysis since we have $\langle r\rangle_{21}^{c\bar{c}}\sim(0.5\ {\rm GeV})^{-1}$. However we may attempt a phenomenological calculation which mimicks the theoretical one done just before; using

Eq. (3.2) with the appropriate changes, i.e $m = m_c$, $n_f = 3$ and replacing $\alpha_s \to \alpha_c^{\rm eff}$, we get perfect agreement with the experimental fine splittings for $\alpha_c^{\rm eff} = 0.86$. The external nonperturbative term is reasonably small and, although $\alpha_c^{\rm eff}$ is large, one can still interpret it as an *effective* coupling into which are lumped internal corrections and higher order perturbative ones. Using then the appropriate modification of Eq. (3.3)

$$\Delta_{{\rm HF},c} = \frac{5}{24}\left(\frac{\beta_0}{2} - \frac{21}{4}\right) C_F \alpha_c^{\rm eff} \Delta_{21}^{\rm th} + \frac{976}{1053} \frac{\pi\langle\alpha_s G^2\rangle}{m^3 (C_F \tilde{\alpha}_c^{\rm eff})^2}\,, \tag{4.4}$$

we get

$$\Delta_{{\rm HF},c} = -2.5 \pm 2.5 \text{ MeV}\,. \tag{4.5}$$

(the error being that in $\langle\alpha_s G^2\rangle$). It is indeed remarkable that the experimental value $\Delta_{{\rm HF},c}^{\rm expt} = -0.9$ MeV falls within the range of the previous prediction. Another noteworthy feature is the role played by the gluon condensate in obtaining (4.4). If we had set $\langle\alpha_s G^2\rangle = 0$ we would have obtained

$$\Delta_{{\rm HF},c} = -9.2 \text{ MeV}$$

widely off experiment.[1] This strongly suggests that the system of Eqs. (3.2) and (3.3) which as we have just seen works reasonably well even for $c\bar{c}$ bound states, can be trusted to provide a reliable description of the $n = 2$, $l = 1$ fine and hyperfine splittings of bottomium.

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[1] In phenomenological papers agreement of $\Delta_{{\rm HF},c}$ with experiment is obtained at the cost of using *different* values of α_s for fine and hyperfine splittings, or extra phenomenological LS or T interactions, or both; see for example Ref. [6] and work quoted there.

Figure Captions.

Fig. 1.- Experimental and Theoretical (from Eq. (3.2)) fine splittings.

Fig. 2.- Hyperfine splitting.

continuous line: central value for $\langle \alpha_s G^2 \rangle$

shaded area: varying $\langle \alpha_s G^2 \rangle$ between its bounds

dotted line: neglecting gluon condensate.

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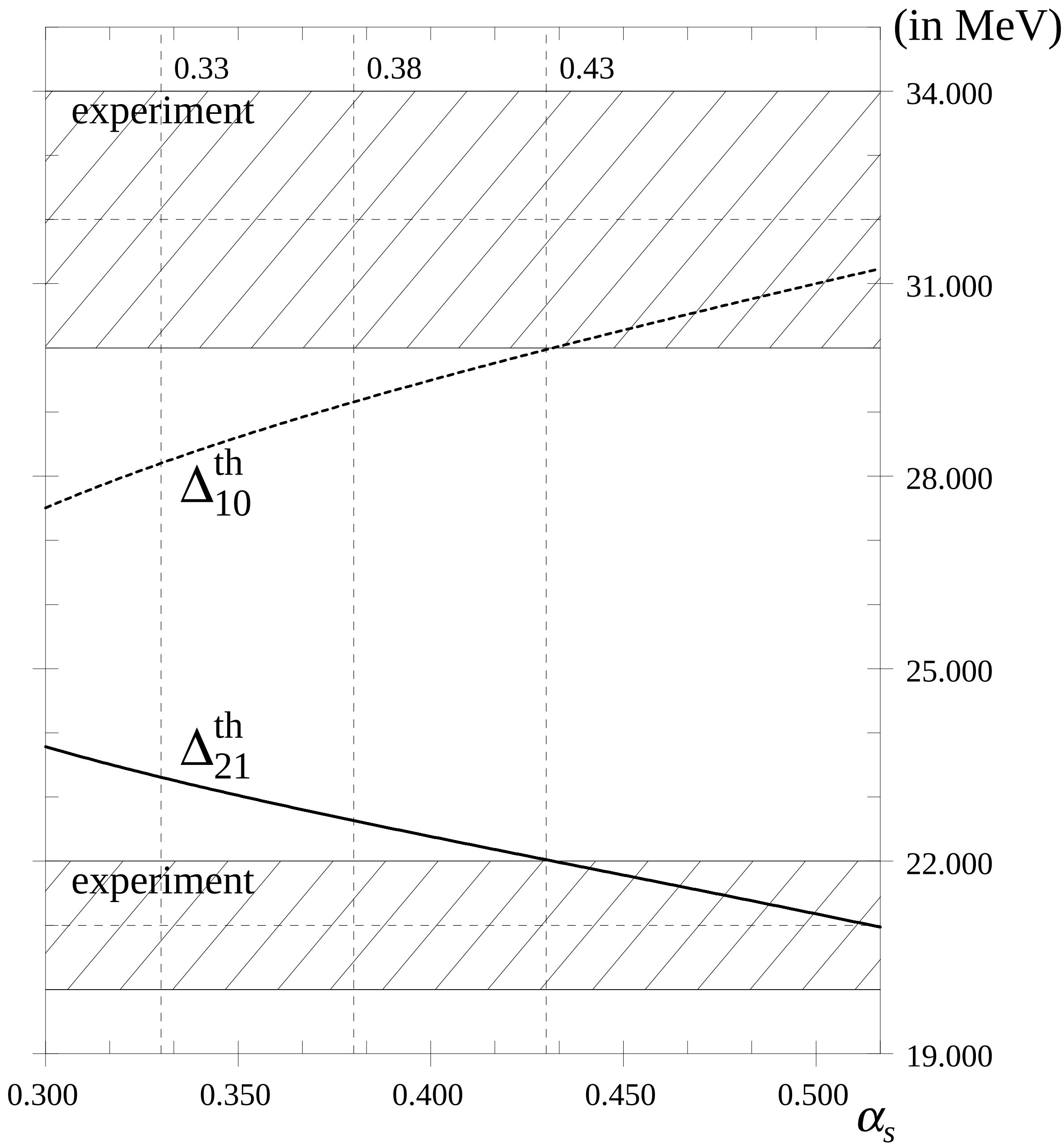
(in MeV)
0.33
0.38
0.43
34.000
31.000
28.000
25.000
22.000
19.000
experiment
experiment
Δ_{10}^{th}
Δ_{21}^{th}
0.300
0.350
0.400
0.450
0.500
α_s

Fig. 1

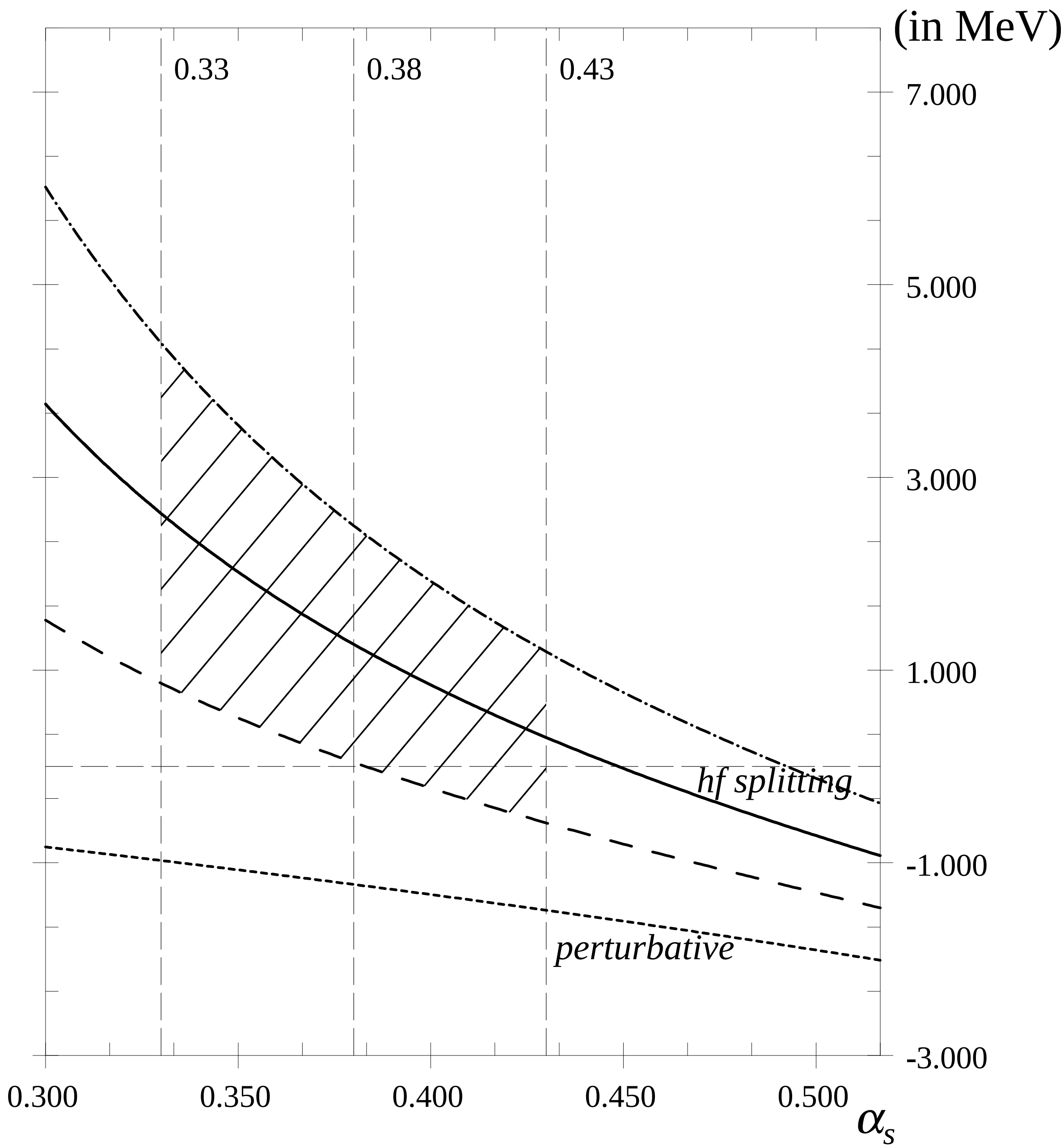
(in MeV)
0.33
0.38
0.43
7.000
5.000
3.000
1.000
-1.000
-3.000
hf splitting
perturbative
0.300
0.350
0.400
0.450
0.500
α_s

Fig. 2